Examination Number:

Set:

Shore

Year 12
HSC Assessment Task 5 - Trial HSC
16th August 2013

Mathematics Extension 1

General Instructions

- Reading time − 5 minutes
- Working time − 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this question paper
- Answer Questions 1–10 on the Multiple Choice Answer Sheet provided
- Start each of Questions 11–14 in a new writing booklet
- Show all necessary working in Questions 11–14
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks – 70

Section I Pages 3–6

10 marks

- Attempt Questions 1−10
- Allow about 15 minutes for this section

Section II Pages 7–12

60 marks

- Attempt Questions 11−14
- Allow about 1 hour 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple Choice Answer Sheet for Questions 1–10.

1 The point P divides the interval OQ, where O is the origin and Q is the point (3, 7), externally in the ratio 5 : 4. What are the coordinates of P?

(A) (–15, –35)

(B) (–12, –28)

(C) (12, 28)

(D) (15, 35)

2 What is the exact value of $\int_0^{\pi} \cos^2 x\,dx$?

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

3 The function $f(x) = x^2 - 17$ has a zero near x_0. Using Newton's Method, which expression gives a better approximation for the zero of the function?

(A) $x_0 - \frac{x_0^{\,2} - 17}{2x_0}$

(B) $x_0 - \frac{2x_0}{x_0^{\,2} - 17}$

(C) $\frac{x_0^{\,2} - 17}{2x_0} - x_0$

(D) $\frac{2x_0}{x_0^{\,2} - 17} - x_0$

4 The equation $2x^3 + 5x^2 - 1 = 0$ has roots $-\frac{1}{2}, \sqrt{2} - 1$ and α.

What is the value of α?

(A) $-\sqrt{2} - 1$

(B) $-\sqrt{2} + 1$

(C) $\sqrt{2} - 1$

(D) $\sqrt{2} + 1$

5 What is the constant term in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{12}$?

(A) –1760

(B) –220

(C) 220

(D) 1760

6 Which of the following is not a solution of $2\cos^2 x - 3\cos x + 1 = 0$?

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) 2π

7 The line $y = 2 - x$ and the curve $y = x^3 + 4$ intersect at the point (–1, 3). Which of the following is the size in radians of the acute angle between these curves at the point (–1, 3)?

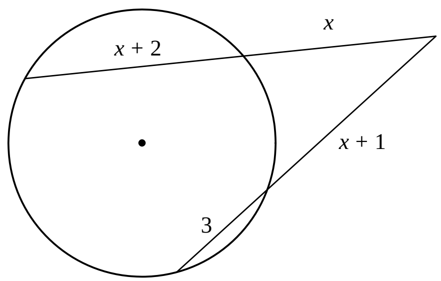
(A) $\tan^{-1}\frac{2}{5}$

(B) $\tan^{-1}\frac{1}{2}$

(C) $\tan^{-1}1$

(D) $\tan^{-1}2$

8 Two secants from an external point cut off intervals on a circle as shown below.



What is the value of x?

(A) $\frac{1 + \sqrt{14}}{2}$

(B) 4

(C) $\frac{-3 + \sqrt{73}}{4}$

(D) 5

9 A particle is moving in simple harmonic motion according to the equation $\ddot{x} = -9x + 3$.
Which of the following gives the centre of motion, x_0, and period, T?

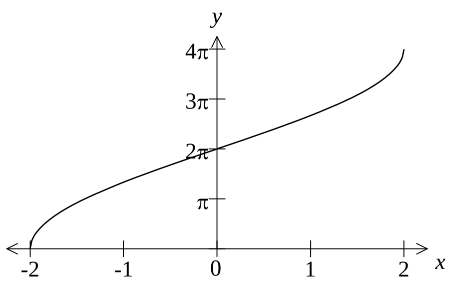
(A) $x_0 = 3, T = \frac{2\pi}{9}$

(B) $x_0 = 3, T = \frac{2\pi}{3}$

(C) $x_0 = \frac{1}{3}, T = \frac{2\pi}{9}$

(D) $x_0 = \frac{1}{3}, T = \frac{2\pi}{3}$

10 The graph below shows a function in the form $y = a\cos^{-1}(bx)$.



What are the values of a and b?

(A) $a = 4, b = \frac{1}{2}$

(B) $a = \frac{1}{4}, b = -\frac{1}{2}$

(C) $a = 4, b = -\frac{1}{2}$

(D) $a = \frac{1}{4}, b = \frac{1}{2}$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Start each of Questions 11–14 in a new writing booklet.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve $\frac{x+1}{x-1} > 2$. **3**

(b) Evaluate $\int_{-3}^{3} \frac{1}{\sqrt{9-x^2}}\,dx$. **2**

(c) Find $\frac{d}{dx}\left(\cos^{-1} e^x\right)$. **2**

(d) Use the substitution $u = x^2 + 2$ to find $\int_0^1 x(x^2+2)^5\,dx$. **3**

(e) Find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = 2\sec\frac{x}{3}$, the x-axis and the lines $x = 0$ and $x = \pi$ is rotated about the x-axis. **3**

(f) Evaluate $\lim_{x\to 0} \frac{x^2}{\sin^2 2x}$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) A roast chicken, which is initially at a temperature of 220°C, is removed from an oven and left to cool on a bench at a constant temperature of 20°C. The cooling rate of the chicken is proportional to the difference between the temperature of the bench and the temperature, T, of the chicken. That is, T satisfies the equation

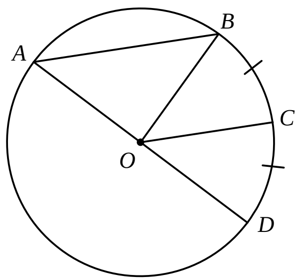
$$\frac{dT}{dt} = -k(T - 20),$$

where t is the number of minutes after the chicken has been placed on the bench.

(i) Show that $T = 20 + Ae^{-kt}$ satisfies this equation. **1**

(ii) The temperature of the chicken is 60°C after 1 hour. Find the temperature of the chicken, to the nearest degree, after 2 hours. **3**

(b) In the diagram below, the points A, B, C and D are concyclic. The point O is the centre of the circle, AD is a diameter of the circle and the arc lengths BC and CD are equal. Show that $AB \parallel OC$. **3**



Question 12 continues on the following page

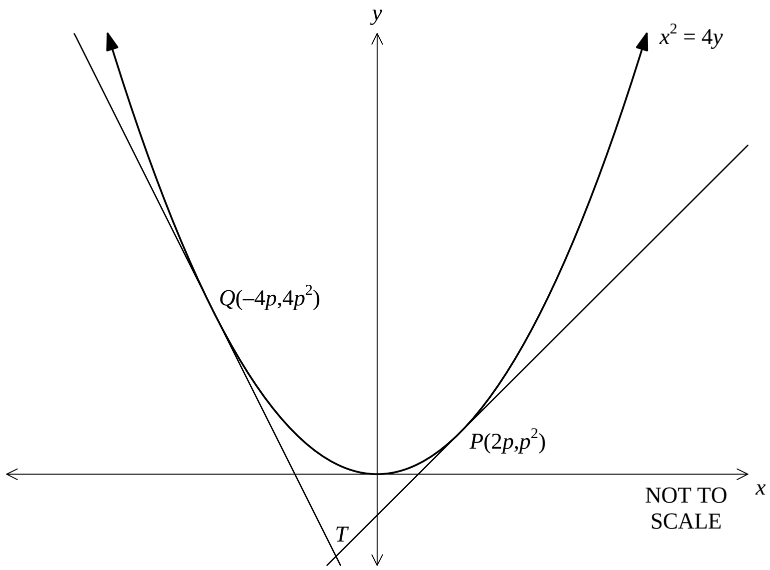
Question 12 continued

(c) Consider the function $f(x) = \sqrt{x} - 1$.

(i) State the domain and range of $y = f(x)$. **1**

(ii) Find an expression for $f^{-1}(x)$. **2**

(d) The diagram shows the parabola $x^2 = 4y$. The points $P(2p, p^2)$ and $Q(-4p, 4p^2)$ lie on the parabola. The tangents at P and Q intersect at T.



(i) Show that the equation of the tangent at P is given by **2**

$$y = px - p^2.$$

(ii) Write down the equation of the tangent at Q, and find the coordinates of the point T in terms of p. **2**

(iii) Find the Cartesian equation of the locus of T. **1**

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) A particle moves in a straight line. Its displacement, x metres, after t seconds is given by

$$x = \cos(5t) - \sin(5t).$$

(i) Show that the particle is moving in simple harmonic motion by showing that $\ddot{x} = -n^2 x$. **2**

(ii) Express the displacement in the form $x = R\sin(5t - \alpha)$, where $0 < \alpha < \frac{\pi}{2}$. **2**

(b) A particle moves in a straight line along the x-axis so that its acceleration is given by $\ddot{x} = \frac{1}{4 + x^2}$ where x is the displacement from the origin.

Initially the particle is at rest at the origin.

(i) Find v^2 as a function of x. **2**

(ii) Explain why v is always positive for $t > 0$. **1**

(iii) Find the velocity as $x \to \infty$. **1**

(c) Find the greatest coefficient in the expansion of $(2x + 1)^{18}$. **3**

(d) The binomial theorem states that **4**

$$(1 + x)^n = \sum_{k=0}^{n} {}^nC_k x^k .$$

By differentiating both sides of this identity twice with respect to x, show that

$$2^{n-2} - 1 = \sum_{k=2}^{n-1} {}^nC_k \frac{k(k-1)}{n(n-1)}, \qquad n \ge 3 .$$

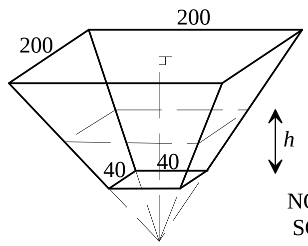
Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Use mathematical induction to prove that **3**

$$\sum_{j=2}^{n} \ln\left(\frac{j-1}{j+1}\right) = \ln\left(\frac{2}{n(n+1)}\right), \qquad n \ge 2 .$$

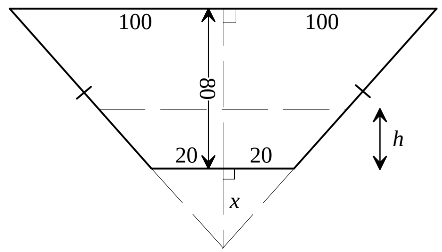
(b) A frustum is a pyramid with its top cut off. Water is being poured into a container in the shape of an inverted right square frustum, as shown below, at a rate of 10 L/s.

Container



NOT TO SCALE

Cross-section



The vertical cross-section of the container has the dimensions (in centimetres) shown above.

(i) Find the value of x. **1**

(ii) Let the depth of water in the container be h centimetres. Show that the volume, V, in cm^3 of water in the container is given by **2**

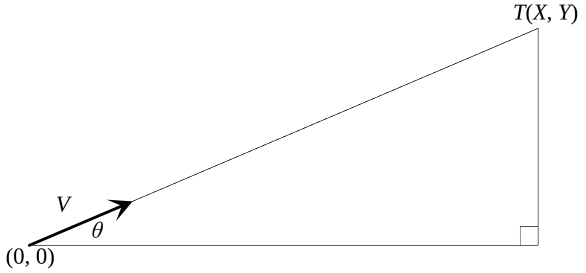
$$V = \frac{4}{3}(h^3 + 60h^2 + 1200h).$$

(iii) How quickly is the depth increasing when the container has been filled to half its height? **3**

Question 14 continues on the following page

Question 14 continued

(c) A ball launcher placed at the origin is aimed at a target, T, X metres away horizontally and Y metres above its launch point, as shown in the diagram below. The initial velocity of the ball is V m/s.



Assume that at time t seconds after the launcher is activated the location of the ball is given by

$$x = Vt\cos\theta$$
$$y = Vt\sin\theta - 5t^2\ .$$

(i) Show that the equation of the path of the ball is given by **2**

$$y = x\left(\frac{Y}{X}\right) - \frac{5x^2}{V^2}\left(1 + \frac{Y^2}{X^2}\right)\ .$$

(ii) Find an expression for V in terms of X and Y that will cause the ball to land directly below the target. **2**

(iii) The initial velocity of the launcher is 20 metres per second. Describe the locus of the target, T, that will allow the ball to land directly below the target. **2**

END OF PAPER

Section I

1. D
2. C
3. A
4. A
5. A
6. B
7. D
8. B
9. D
10. C

1. $P = \left(\frac{4(0) + -5(3)}{4 + -5}, \frac{4(0) + -5(7)}{4 + -5} \right)$

$= (15, 35)$.

2. $\int_0^{\pi} \cos^2 x \, dx = \int_0^{\pi} \frac{1}{2}(\cos 2x + 1)\, dx$

$= \frac{1}{2}\left[\frac{\sin 2x}{2} + x \right]$

$= \frac{1}{2}\left(\left(\frac{\sin 2\pi}{2} + \pi \right) - (\sin 0 - 0) \right)$

$= 0 + \frac{\pi}{2}$

$= \frac{\pi}{2}.$

3. $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{x_0^2 - 17}{2x_0}$

4. $\alpha + \sqrt{2} - 1 - \frac{1}{2} = \frac{-5}{2}.$

$\alpha + \sqrt{2} - 1 = -2$

$\alpha = -\sqrt{2} - 1$

5. $(2x^3 - \frac{1}{x})^{12} = \sum_{k=0}^{12} {}^{12}C_k (2x^3)^{12-k} (-x^{-1})^k$

$= \sum_{k=0}^{12} {}^{12}C_k \, 2^{12-k} x^{36-3k} (-1)^k x^{-k}$

Constant term when $36 - 4k = 0$

$k = 9.$

5. cont.

constant term $= {}^{12}C_9 \, 2^3 (-1)^9$

$= -1760.$

6. $2\cos^2 x - 3\cos x + 1 = 0.$ let $u = \cos x$.

$2u^2 - 3u + 1 = 0$

$(2u - 1)(u - 1) = 0.$

$\therefore u = 1$ or $u = \frac{1}{2}.$

$\cos x = 1$ or $\cos x = \frac{1}{2}$

$\therefore x = 0, 2\pi$ or $x = \frac{\pi}{3}, \frac{5\pi}{3}.$

$\therefore \frac{\pi}{6}$ is not a solution

7. From $y = 2 - x$, $m_1 = -1$

From $y = x^3 + 4$

$y' = 3x^2$

$= 3(-1)^2$ when $x = -1$.

$\therefore m_2 = 3.$

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$= \left| \frac{-1 - 3}{1 - 3} \right|$

$= 2$

$\therefore \theta = \tan^{-1}(2).$

8. $x(2x+2) = (x+1)(x+4)$

$2x(x+1) = (x+1)(x+4)$

$(x+1)(2x - (x+4)) = 0$

$(x+1)(x-4) = 0.$

$\therefore x = 4, -1.$

But $x > 0$

$\therefore x = 4.$

9. $\ddot{x} = -9x + 3$

$= -9(x - \frac{1}{3})$

$\therefore x_0 = \frac{1}{3}$, $T = \frac{2\pi}{3}.$

10. $a = 4$, $b = -\frac{1}{2}.$

Section II

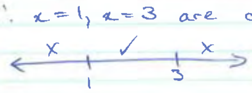
Q11

a) Solve $\frac{x+1}{x-1} = 2$, $x \neq 1$

$x + 1 = 2x - 2$

$x = 3$

$\therefore x = 1, x = 3$ are critical points.



$\therefore$ solution is $1 < x < 3.$

b) $\int_{-3}^{3} \frac{1}{\sqrt{9 - x^2}}\, dx = \left[\sin^{-1}\left(\frac{x}{3}\right) \right]_{-3}^{3}$

$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$

$= \pi.$

c) $\frac{d}{dx}(\cos^{-1} e^x) = \frac{-1}{\sqrt{1 - (e^x)^2}} \cdot e^x$

$= \frac{-e^x}{\sqrt{1 - e^{2x}}}.$

d) $\int_0^1 x(x^2+2)^5 \, dx = \int_2^3 \frac{u^5}{2}\, du$

let $u = x^2 + 2$

$\frac{du}{dx} = 2x$

$x\,dx = \frac{du}{2}$

When $x = 0$, $u = 2$

When $x = 1$, $u = 3$

$= \left[\frac{u^6}{12} \right]_2^3$

$= \frac{3^6}{12} - \frac{2^6}{12}$

$= \frac{665}{12}$

Q11 cont.

e) $V = \pi \int_0^{\pi} 4\sec^2\left(\frac{x}{3}\right) dx$

$= 4\pi\left[3\tan\left(\frac{x}{3}\right)\right]_0^{\pi}$

$= 12\pi\left(\tan\frac{\pi}{3} - \tan 0\right)$

$= 12\pi\sqrt{3}$ units^3

f) $\lim_{x \to 0} \frac{x^2}{\sin^2 2x} = \lim_{x \to 0} \left(\frac{2x}{\sin 2x}\right)^2 \times \frac{1}{4}$

$= 1 \times 1 \times \frac{1}{4}$

$= \frac{1}{4}$.

Q12

a)(i) $T = 20 + Ae^{-kt}$ $\Rightarrow$ $T - 20 = Ae^{-kt}$

$\frac{dT}{dt} = -kAe^{-kt}$

$= -k(T-20)$

(ii) When $t = 60$, $T = 60°$

when $t = 0$, $T = 220°$.

So $220 = 20 + Ae^{-0}$

$\therefore A = 200$

So $T = 20 + 200e^{-kt}$

And $60 = 20 + 200e^{-60k}$

$\frac{40}{200} = e^{-60k}$

$-\ln 5 = -60k$

$k = \frac{\ln 5}{60}$.

Q12 cont.

a) (ii) cont.

When $t = 120$

$T = 20 + 200e^{-120 \times \frac{\ln 5}{60}}$

$= 20 + 200e^{-2\ln 5}$

$= 28°C$.

b) Let $\angle COD = x°$

$\therefore \angle BOC = x°$ (equal arcs (BC, CD) subtend equal angles at the centre)

So $\angle BOD = 2x°$ ($\angle BOC + \angle COD$)

$\therefore \angle BAD = x°$ (angle at circumference half angle at centre on same arc).

$\therefore \angle ABO = x°$ (base angles of isosceles $\triangle ABO$, $AO = BO$; equal radii)

So $\angle ABO = \angle COB$

$\therefore AB \parallel OC$ (equal alternate angles).

c) $f(x) = \sqrt{x} - 1$.

(i) domain: $x \geq 0$

range: $y \geq -1$.

(ii) $y = \sqrt{x} - 1$

has inverse

$x = \sqrt{y} - 1$

$x + 1 = \sqrt{y}$

$y = (x+1)^2$, $x \geq -1$.

$\therefore f^{-1}(x) = (x+1)^2$, $x \geq -1$.

Q12 cont.

d) (i) $y = \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{2x}{4}$

$= \frac{x}{2}$.

Tangent at P: $y - p^2 = \frac{2p}{2}(x - 2p)$

$y = px - 2p^2 + p^2$

$y = px - p^2$

(ii) Tangent at Q: $y - (2p)^2 = -2p(x + 4p)$

$y - 4p^2 = -2px - 8p^2$

$y = -2px - 4p^2$

At T: $px - p^2 = -2px - 4p^2$

$3px + 3p^2 = 0$

$3p(x + p) = 0$.

$\therefore x = -p$, $y = -2p^2$.

So $T = (-p, -2p^2)$.

(iii) $p = -x$

$y = -2(-x)^2$

$y = -2x^2$ is the locus of T.

Q13

a) $x = \cos 5t - \sin 5t$

(i) $\dot{x} = -5\sin 5t - 5\cos 5t$

$\ddot{x} = -25\cos 5t + 25\sin 5t$

$= -25(\cos 5t - \sin 5t)$

$= -5^2 x$

which is in the form $\ddot{x} = -n^2 x$, where $n = 5$.

(ii) Let $\cos 5t - \sin 5t = R\sin(5t - \alpha)$

$= R\sin 5t\cos\alpha - R\cos 5t\sin\alpha$

$\therefore -R\sin\alpha = 1$

$R\cos\alpha = -1$

So $-\tan\alpha = -1$

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$. $(0 \leq \alpha \leq \frac{\pi}{2})$.

Sub. $\alpha = \frac{\pi}{4}$ into $R\cos\alpha = -1$

$\therefore R = \frac{-1}{\cos\frac{\pi}{4}}$

$= \frac{-1}{\frac{1}{\sqrt{2}}} = -\sqrt{2}$.

$\therefore x = -\sqrt{2}\sin\left(5t - \frac{\pi}{4}\right)$

b) $\ddot{x} = \frac{1}{4 + x^2}$

(i) $\frac{d\left(\frac{1}{2}v^2\right)}{dx} = \frac{1}{4 + x^2}$

$\frac{1}{2}v^2 = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

$v^2 = \tan^{-1}\left(\frac{x}{2}\right) + K$.

Q13 (cont.)

b) (i) cont.

When $x = 0$, $v = 0$.

So $0 = \tan^{-1}(0) + K$.

$\therefore K = 0$.

$\therefore v^2 = \tan^{-1}\left(\frac{x}{2}\right)$

(ii) $\ddot{x} = \frac{1}{4+x^2}$

> 0 for all x as $x^2 \geq 0$ for all x.

If the particle begins at rest at the origin and its acceleration is always positive, it will always be moving in the direction of the positive x-axis.

i.e. $v > 0$.

(iii) $\lim_{x \to \infty} v = \lim_{x \to \infty} \sqrt{\tan^{-1}\left(\frac{x}{2}\right)}$

$= \sqrt{\frac{\pi}{2}}$.

c) $(2x+1)^{18} = \sum_{k=0}^{18} {}^{18}C_k \, (2x)^{18-k} \, 1^k$

$= \sum_{k=0}^{18} {}^{18}C_k \, 2^{18-k} x^{18-k}$

Q13 (cont.)

c) cont.

We want $\dfrac{{}^{18}C_{k+1}\, 2^{18-k-1}}{{}^{18}C_k \, 2^{18-k}} > 1$

$\dfrac{18!}{(18-k-1)!\,(k+1)!} \times \dfrac{(18-k)!\,k!}{18!} \times \frac{1}{2} > 1$.

$\dfrac{18-k}{2(k+1)} > 1$

$18 - k > 2k + 2$

$16 > 3k$

$k < \frac{16}{3}$

$\therefore k = 5$ is the greatest such integer.

So ${}^{18}C_6 \, 2^{12} = 76038144$ is the greatest coefficient.

d) $(1+x)^n = \sum_{k=0}^{n} {}^nC_k \, x^k$

Differentiating: $n(1+x)^{n-1} = \sum_{k=0}^{n} {}^nC_k \, k x^{k-1}$

Differentiating again: $n(n-1)(1+x)^{n-2} = \sum_{k=0}^{n} {}^nC_k \, k(k-1) x^{k-2}$

Let $x = 1$: $n(n-1)2^{n-2} = \sum_{k=0}^{n} {}^nC_k \, k(k-1)$

So $2^{n-2} = 0 + 0 + \sum_{k=2}^{n-1} {}^nC_k \dfrac{k(k-1)}{n(n-1)} + 1$.

(↑ $\frac{0 \times -1}{n(n-1)}$, ↑ $\frac{1 \times 0}{n(n-1)}$, ↑ ${}^nC_n \times \frac{n(n-1)}{n(n-1)}$)

$\therefore 2^{n-2} - 1 = \sum_{k=2}^{n-1} {}^nC_k \dfrac{k(k-1)}{n(n-1)}$.

Q14

a) Prove true for $n = 2$.

LHS $= \sum_{j=2}^{2} \ln\left(\frac{j-1}{j+1}\right) = \ln\left(\frac{1}{3}\right)$

RHS $= \ln\left(\frac{2}{2 \times 3}\right) = \ln\left(\frac{1}{3}\right)$

LHS = RHS. $\therefore$ true for $n = 2$.

~~Pro~~ Assume true for $n = k$.

i.e. $\sum_{j=2}^{k} \ln\left(\frac{j-1}{j+1}\right) = \ln\left(\frac{2}{k(k+1)}\right)$

Prove true for $n = k+1$.

i.e. need to show $\sum_{j=2}^{k+1} \ln\left(\frac{j-1}{j+1}\right) = \ln\left(\frac{2}{(k+1)(k+2)}\right)$

LHS $= \sum_{j=2}^{k+1} \ln\left(\frac{j-1}{j+1}\right)$

$= \sum_{j=2}^{k} \ln\left(\frac{j-1}{j+1}\right) + \ln\left(\frac{k}{k+2}\right)$

$= \ln\left(\frac{2}{k(k+1)}\right) + \ln\left(\frac{k}{k+2}\right)$ by the inductive hypothesis.

$= \ln\left(\frac{2k}{k(k+1)(k+2)}\right)$

$= \ln\left(\frac{2}{(k+1)(k+2)}\right)$

= RHS.

$\therefore$ true for $n = k+1$.

$\therefore$ by the principle of mathematical induction, true for all $n \geq 2$.

Q14 cont.

b) (i) Using similar triangles.

$\frac{x}{20} = \frac{x+80}{100}$

$100x = 20x + 1600$

$80x = 1600$

$x = 20$.

(ii) $V = \frac{1}{3} A_{top} h_{top} - \frac{1}{3} A_{bottom} h_{bottom}$

$= \frac{1}{3}\left(2(h+20)\right)^2 (h+20) - \frac{1}{3}(2 \times 20)^2 (20)$

$= \frac{4}{3}(h+20)^3 - \frac{4}{3}(20)^3$

$= \frac{4}{3}(h^3 + 60h^2 + 1200h + 8000) - \frac{4}{3}(8000)$

$= \frac{4}{3}(h^3 + 60h^2 + 1200h)$

(iii) $\frac{dV}{dh} = \frac{4}{3}(3h^2 + 120h + 1200)$

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

$= 10000 \text{ cm}^3/\text{s} \times \frac{3}{4}\left(\frac{1}{3 \times 40^2 + 120 \times 40 + 1200}\right) \text{ cm/cm}^3$

$= 10000 \times \frac{3}{4 \times 10800}$

$= 0.69\dot{4}$ cm/s.

Q14

c) (i) $x = Vt\cos\theta$

$t = \dfrac{x}{V\cos\theta}$

$\therefore y = V\left(\dfrac{x}{V\cos\theta}\right)\sin\theta - 5\left(\dfrac{x^2}{V^2\cos^2\theta}\right)$

$= x\tan\theta - \dfrac{5x^2}{V^2}\left(\dfrac{1}{\cos^2\theta}\right)$

$= x\left(\dfrac{Y}{X}\right) - \dfrac{5x^2}{V^2}\left(\dfrac{1}{\left(\dfrac{X}{\sqrt{X^2+Y^2}}\right)^2}\right)$

$= x\left(\dfrac{Y}{X}\right) - \dfrac{5x^2}{V^2}\left(\dfrac{X^2+Y^2}{X^2}\right)$

$y = x\left(\dfrac{Y}{X}\right) - \dfrac{5x^2}{V^2}\left(1+\dfrac{Y^2}{X^2}\right)$ ■

(ii) Directly below target $\Rightarrow x = X,\ y = 0.$

$0 = X\left(\dfrac{Y}{X}\right) - \dfrac{5X^2}{V^2}\left(1+\dfrac{Y^2}{X^2}\right)$

$0 = Y - \dfrac{5}{V^2}(X^2+Y^2)$

$\dfrac{5}{V^2}(X^2+Y^2) = Y$

$V^2 = \dfrac{5}{Y}(X^2+Y^2)$

$V = \sqrt{\dfrac{5}{Y}(X^2+Y^2)}$, as $V > 0$.

Q14

c) (iii) $20 = \sqrt{\dfrac{5}{Y}(X^2+Y^2)}$

$400 = \dfrac{5}{Y}(X^2+Y^2)$

$400Y = 5(X^2+Y^2)$

$80Y = X^2+Y^2$

$X^2+Y^2-80Y = 0$

$X^2+Y^2-80Y+1600 = 1600$

$X^2+(Y-40)^2 = 40^2$

$\therefore$ the locus of T is a circle centred at $(0,40)$, radius 40.